A COMPUTATIONAL MODEL OF THE BAROQUE TRUMPET AND MUTE

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Unlike modern trumpet mutes, the Baroque trumpet mute raises the playing pitch of the instrument. This pitch change is the subject of this paper.

The Baroque mute is turned from hardwood and fits snugly into the bell. Air and sound pass through a relatively small-diameter hole bored through the center of the mute. As discussed by Keller and Smithers in previous papers,[1] musical examples and contemporary written material indicate that the pitch rise is a whole tone. However, trials with extant mutes have shown a pitch rise of only a semitone. In this paper, we report on a computational model of trumpet and mute which allows the calculation of the air-column resonance frequencies that determine the playing frequencies of the instrument.

The Experiment

Imagine that we have a trumpet and mouthpiece and that the mouthpiece is sealed at the rim, much as it is by the player's lips. Imagine further that a small volume of air is injected into the mouthpiece cup through a tiny hole. This air is periodically pumped in and out of the mouthpiece in a pure tone at a single frequency, just as if it were controlled by a sine-wave electronic oscillator. Suppose also that we have a small microphone inside the mouthpiece cup with which we measure the sound pressure produced there by the injected air. (This sound pressure will be a small sine-wave fluctuation in pressure superimposed on the much larger steady atmospheric pressure; the amplitude of the sound pressure will be proportional to the amplitude of the injected signal.)

If we now slowly change the frequency of the injected air signal, while maintaining its flow rate constant, we will find that the sound pressure inside the mouthpiece will depend on the frequency, being larger at some frequencies, smaller at others. Those frequencies where the pressure response is large are the resonances where the instrument will help the player to sound the trumpet. At frequencies where the response is small, the instrument will actually hinder the vibration of the lips, compared with simply buzzing on the mouthpiece alone.

The *ratio* of the sound pressure produced by the injected air to the rate of flow of the air is called the *acoustic impedance*. Because the sound pressure is proportional to the injected test signal, this ratio depends only on the properties of the instrument and is independent of the amplitude of the test signal. Thus the frequencies of the peaks in acoustic impedance "looking into the mouthpiece" will, subject to some interpretation, tell us quite a lot about the playing frequencies of the instrument.

This kind of experiment and the reasons why lip vibration is aided by high impedance are described in some detail in the excellent book by Benade.[2] In this paper, the "experiment" was carried out on a computer simulating the trumpet and mouthpiece, with and without a mute. Some of the mathematical details of the computer model are given in Appendix A.

Instrument Dimensions

Early Baroque trumpets (sixteenth- and early seventeenth-century instruments) have a bell contour distinctly different from later instruments. Therefore, two instruments were modelled: a 1632 trumpet by Hanns Hainlein and a 1746 trumpet by Johann Leonhart Ehe III. Dimensions were taken from drawings supplied by Robert Barclay. Numerical values used in the calculations for the mouthpiece, trumpets, and mute are given in Appendix B.

The instruments were designed for a playing pitch of A 415 Hz. Playing pitch during the Baroque was rather variable from time to time and place to place. As a *de facto* standard, A 415 is commonly used today for Baroque and Classical performance. It has the great virtue of being almost exactly a semitone below the modern pitch standard of A 440, which makes it possible to build keyboard instruments such as portative organs which can be played either at A 415 or at A 440 simply by moving the keyboard one note to the right or left. For the two trumpets, the length of the cylindrical tubing between mouthpiece and bell was initially taken from the drawings. These lengths gave a playing pitch slightly sharper than A 415. Consequently, small adjustments (the computerized equivalent of tuning bits) were made to lower the pitch of the unmuted instruments very close to a scale built on harmonics of a D of 69.2 Hz, corresponding to an A of 415 Hz.

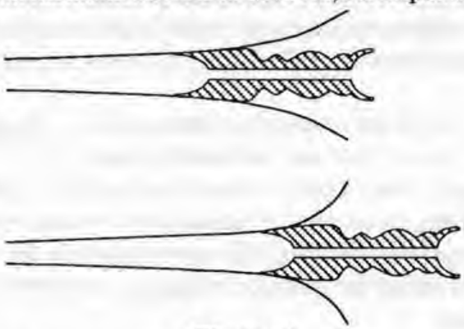

Figure 1

Cross section of the Hainlein bell (above) and the Ehe bell (below) with mute. Note the more gradual flare and larger bell throat of the earlier (Hainlein) bell. The two bells are drawn to the same scale and the rims are shown in the same plane in order to show how much farther the mute is inserted into the Hainlein bell.

The Ehe trumpet has a bell contour very similar to a modern B-flat piston valve instrument, but slightly smaller in diameter throughout. The Hainlein trumpet has a more gradually flaring bell contour which could well be derived from the shape of an animal horn. At the bell rim, the Ehe is about a centimeter larger in diameter, but at distances more than a few centimeters upstream, the Hainlein is the larger, until just before the cylindrical tubing. The Hainlein has a cylindrical bore of 10.5 mm.; the Ehe is slightly larger at 11.0 mm.

Mouthpiece dimensions were taken from a mouthpiece owned by Fred Holmgren.

The mute dimensions used in this study were taken from a mute made and owned by Fred Holmgren. It was turned to fit a trumpet owned by Mr. Holmgren, made by Ronald Collier and similar to the Ehe. It is of the one-sided variety and is turned from maple. Its dimensions are typical of the mutes in the Prague collection. The internal shape is patterned after a drawing by Altenburg.

A given mute will be inserted farther into the Hainlein bell than the Ehe due to its larger throat. The pitch change due to muting is therefore expected to be greater for the Hainlein than for the Ehe. Figure 1 shows a cross section of the mute in the two bells.

Impedance Calculations

Figure 2 shows the *impedance magnification* of the Ehe trumpet; that is, the ratio of its impedance at the mouthpiece to the impedance of an infinitely long cylindrical tube whose diameter is the same as the mouthpiece cup. The frequency scale is logarithmic, so that the spacing between impedance peaks is proportional to the musical interval between them. Note that the height of the resonant peaks is greatest in the vicinity of 900 - 1000 Hz, near the twelfth or thirteenth resonance of the D trumpet. This is due to the mouthpiece. A cup mouthpiece has its own characteristic resonance (the pitch one hears when slapping the rim against the palm of the hand). Below this resonance the mouthpiece magnifies the input impedance of the remainder of the instrument, the greatest magnification occurring near the mouthpiece resonance. This is one of the two major factors influencing the tone quality; the other is the radiation properties of the bell. Those harmonics in the radiated sound lying near 900 - 1000 Hz will be the strongest. For example, if one plays the written middle C, its third harmonic will coincide in frequency with the twelfth resonance of the instrument (if the instrument is properly in tune with itself!) and will be stronger than either the fundamental or the second harmonic.[3]

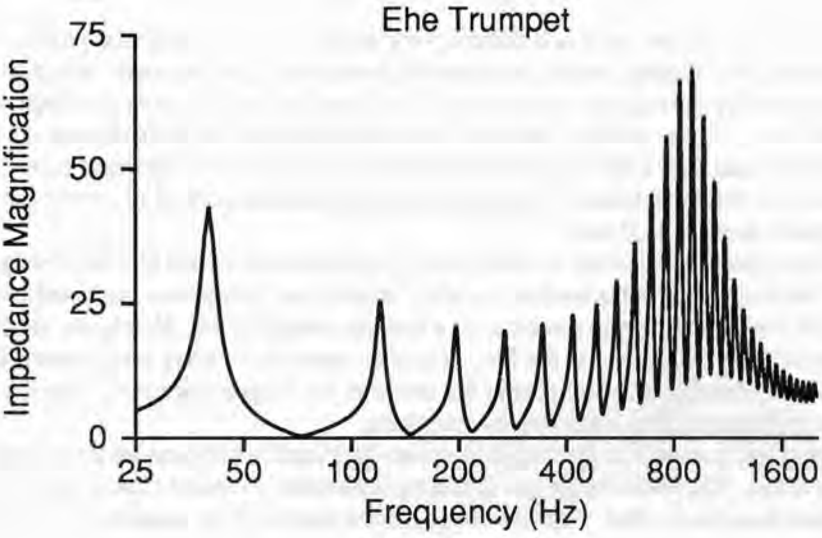

Figure 2
The calculated impedance magnification of the 1746 Ehe trumpet.

Note also that the lowest air-column resonance lies below 50 Hz, well below the nominal 69.2 Hz fundamental of the trumpet. In fact, the first three resonances are substantially lower than the frequencies of the notes the player produces. This is characteristic of all brasses, more so for those like the natural trumpet which have a high percentage of cylindrical tubing than for, say, the flugelhorn and euphonium.

The fact that the player can produce the expected low-register pitches, albeit sometimes with difficulty, without the help of a resonance at the playing frequency is due to an effect discovered by the French physicist Henri Bouasse, termed *sons privilégiés*, or *privileged tones*.[4] The essence of this is that there need not be an air-column resonance near the playing frequency provided there are a sufficient number of resonances close to harmonics of the playing frequency. The privileged-tone effect is responsible for the production of what the hand-horn player terms "factitious tones", notes not in the harmonic series of the instrument, but an octave below "normal" tones.[5] The interaction with higher resonances increases with loudness, so that one normally finds the pitch stability of the lower notes to be better when they are strongly played. It is not widely appreciated that all brass instruments, modern as well as ancient, make use of the privileged-tone effect in the low register. For example, the modern horn player can play a complete harmonic series, right down to the fundamental of the F horn, even though the lowest air-column resonance, which one would like to associate with that note, is about half an octave lower.

Above 1000 Hz, the height of the impedance peaks dies away, both because the magnification produced by the mouthpiece is declining and because the damping, particularly that component caused by radiation from the bell, is increasing.

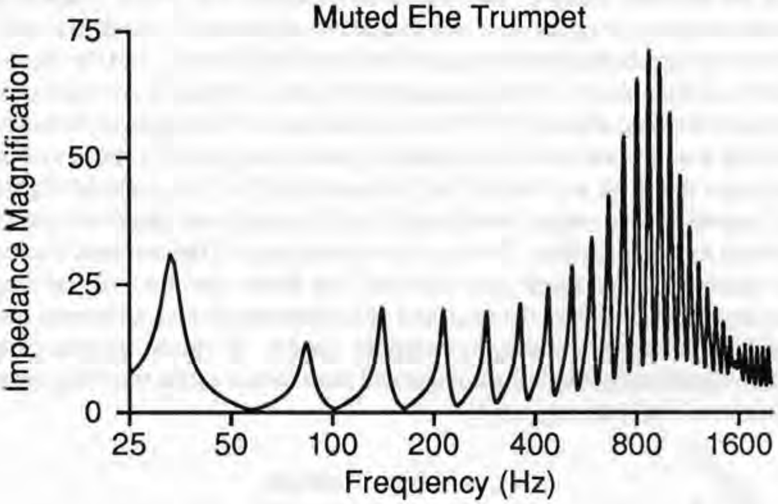

Figure 3
The calculated impedance magnification of the muted 1746 Ehe trumpet.

Next consider the input impedance of the same trumpet muted, shown in Figure 3. Superficially, it appears very similar to the unmuted trumpet. However, there are some important differences. Note that the very highest-frequency impedance peaks (above 1600 Hz) are higher for the muted trumpet because radiation damping has been reduced. Note also a peculiar disruption of the regular pattern of peaks and valleys at about 1600 Hz. This is the frequency at which the hole bored down the center of the mute becomes one-half wavelength long.

All the resonance frequencies have been lowered by the introduction of the mute. The second peak also appears low in height in comparison with its neighbors. Since all resonances have been lowered in frequency by the mute, why then does the player say that the pitch of the instrument has been raised? The answer lies in the fact that the relationship between the frequencies of the peaks has also changed (although this is difficult to see purely by examining the figure). For example, the interval between the fifth and sixth peaks is about a major third, which is the interval one expects to find between the fourth and fifth harmonics of the instrument. It is as though one of the lower-frequency peaks is an intruder and the fifth peak should be considered the fourth, and so on. This is in fact the case, although it is impossible to say which of the peaks is the "extra" one.

The mute itself has a Helmholtz resonance (named after the nineteenth-century German physicist and physiologist) at about 250 Hz, between the third and fourth impedance peaks of the unmuted trumpet. The Helmholtz resonance is a "bottle" resonance, as in the resonance whose frequency one hears when blowing across the neck of a bottle. The hole bored through the body of the mute is the "neck" of the bottle, and the "body" is the cup at the end of the mute with the upstream end closed. The half-wave resonance of the hole through the mute at about 1600 Hz is thus the *second* resonance of the mute. If we consider the trumpet and mute to be separate, rather than parts of a single system, then we can argue that both the fourth (214 Hz) and fifth (288 Hz) peaks of Figure 3 are "fourth" modes of the trumpet. At the fourth peak, the mute looks massive to the trumpet, at the fifth it looks spring-like. Thus the fourth resonance of the unmuted trumpet (269 Hz) has been split into a pair of resonances, one lower than the original frequency because the mute has loaded the open end of the instrument with additional mass, the other higher because the mute has stiffened the system. Of course, to some extent this argument is sophistry because the trumpet and mute form a single vibrating system and should not artificially be separated.

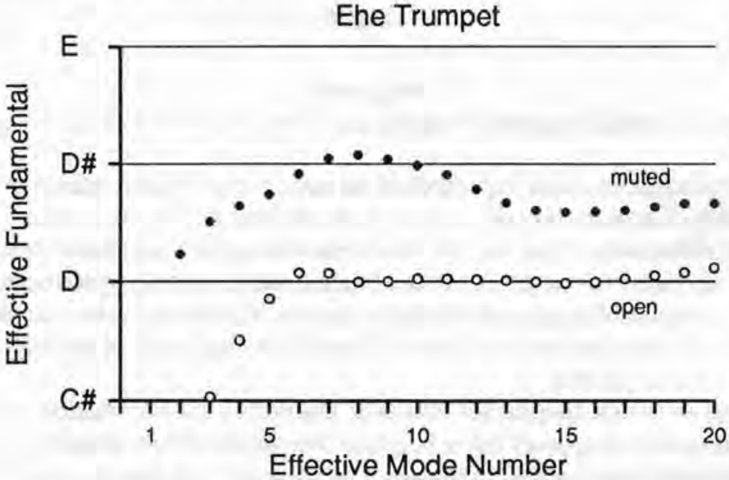

Figure 4
Relative intonation of the 1746 Ehe trumpet.

This "off by one" mode numbering ambiguity lies at the heart of the two-century old disagreement about the pitch change of the fully-stopped horn.[6] For the trumpet player, the mute is either in or out, so the degree of muting is not an issue as it is in hand-stopping. He or she finds a sharpened harmonic series except that there may be some peculiar behavior in the low register (but that is often true for the unmuted natural trumpet as well!).

If one renumbers the peaks of the muted trumpet above the Helmholtz mute resonance, the fifth becoming the fourth, *etc.*, then the resonances from about the "new" fourth peak upwards fit the harmonic series of a sharper instrument reasonably well. Figure 4 shows the resonance frequencies of the open and muted Ehe trumpet. The frequencies have been divided by the "effective mode number" to give an "effective fundamental frequency". The effective mode number for the open trumpet is just the number of the peak; for the muted trumpet, it is one less than the peak number, *i. e.*, the muted modes have been renumbered as decribed above. In such a figure, if all resonances followed the harmonic series exactly, the plotted points would all lie on a horizontal line.

For the open Ehe trumpet, modes from the sixth upwards are very nearly harmonically related. Below the sixth mode, the resonance frequencies are flatter. The third is about a semitone low, and the first and second are off the bottom of the graph.

For the muted Ehe trumpet, the relative intonation is not as good. For the important range between the sixth and twelfth harmonics, the air-column resonance frequencies are very nearly a semitone sharper than for the open instrument. Above this, the resonances are somewhat flatter. A quick check with Fred Holmgren playing two trumpets similar to the Ehe showed a pitch rise of very close to a semitone on the eighth, ninth, and tenth harmonics with the mute modelled here. Above the tenth harmonic, the muted instrument was slightly flatter, as would be expected from Figure 4.

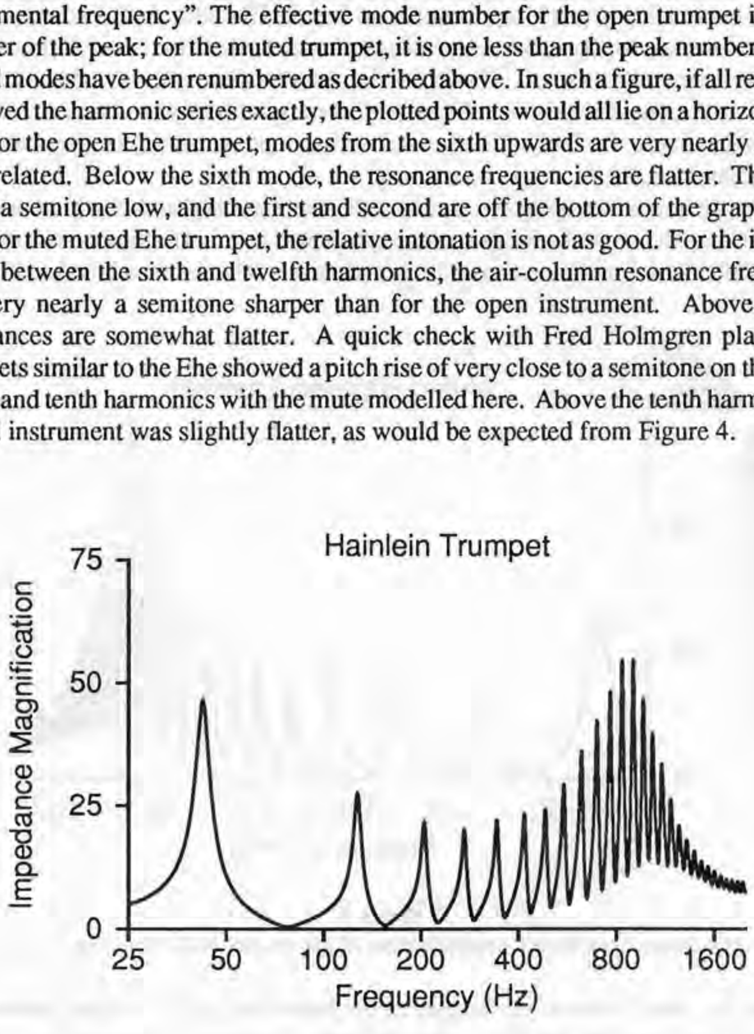

Figure 5
The calculated impedance magnification of the 1632 Hainlein trumpet.

Figures 5 and 6 show the input impedance of the Hainlein trumpet, open and muted, respectively. Again, peaks near 1000 Hz are high but less so than for the Ehe for the unmuted trumpet. This is caused partly by the smaller-bore cylindrical tubing, which introduces more damping, and partly by the bell shape. Any bell radiates very poorly at low frequencies and very well at high frequencies. The difference between our two bells lies in the width of the transition from poor to efficient radiation as frequency increases. The more gradual and somewhat more conical flare of the Hainlein bell begins to radiate more efficiently at lower frequencies than the Ehe bell. As Benade says, "The conical instruments begin to leak sound at lower frequencies than do their flaring cousins having the same bell diameter".[7] The Hainlein's smaller diameter at the rim also means that it does not achieve maximum radiation efficiency until a higher frequency than the Ehe. In other words, the Hainlein bell switches from a poor radiator to a good one more gradually than the Ehe.[8]

Hence, increased damping due to the smaller bore and higher radiation reduces the height of the resonant peaks for the unmuted Hainlein relative to the Ehe in the region near 1000 Hz. Based on this, one would expect the tone of the Hainlein to be less "brassy" than the Ehe.

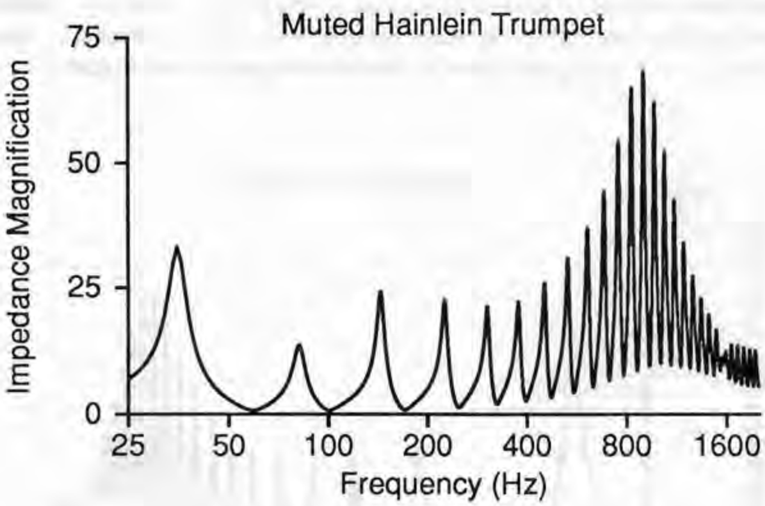

Figure 6
The calculated impedance magnification of the muted 1632 Hainlein trumpet.

With the mute in place, the heights of the impedance peaks for the Hainlein are comparable to those for the Ehe, since radiation damping is now similar for both instruments. As in the Ehe, the anomaly produced by the second resonance of the mute occurs at about 1600 Hz, and the height of the second impedance peak is relatively low.

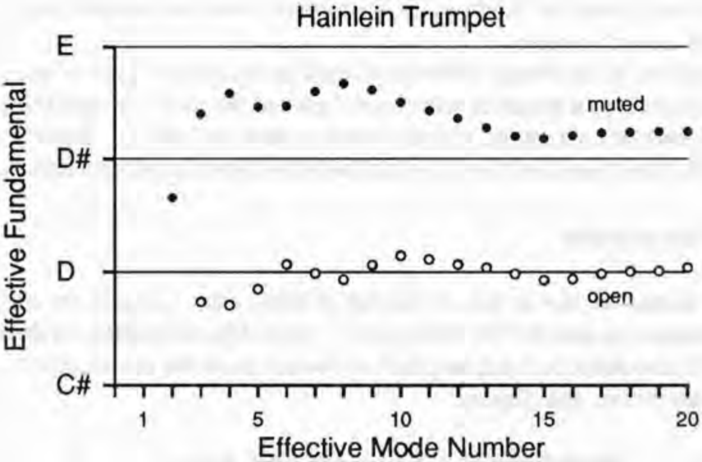

Figure 7
Relative intonation of the 1632 Hainlein trumpet.

Figure 7 shows the normalized resonance frequencies of the Hainlein trumpet. The open trumpet is less well in tune with itself than the Ehe, although the fourth and fifth resonances are less flat compared to the higher modes. The mute indeed sharpens the Hainlein more than the Ehe, but still well short of a whole tone.

One additional calculation was performed in an attempt to sharpen the muted trumpet yet more. The cup at the upstream end of the mute was assumed to be filled, so that the central tube carried through to the end of the mute. The resulting intonation pattern for the Hainlein trumpet was very similar to Figure 7 except that all the muted frequencies were sharper by about one-tenth of a semitone. The additional sharpening is still insufficient for the mute to raise the pitch by a whole tone.

Conclusions

A pitch rise of approximately a semitone on the Ehe is observed in the computer experiment just as it has been by the player. The Hainlein gives a greater sharpening, but still not close enough to a whole tone to be musically useful. It would be desirable to validate this computer model by comparing experimental measurements of impedance and air-column resonance frequencies with those predicted by the model.

The mystery is still unsolved. It might be possible to make a smaller-diameter mute that would produce a whole-tone pitch rise on a D trumpet, particularly on the older bell contour, but no historical examples appear to exist. Such a mute would have to fit the bell at least 22 cm upstream of the bell rim. The longest of the Prague mutes is 15.8 cm overall; a whole-tone mute of that length would be difficult to insert and would need a cord to retrieve it from the depths of the instrument. One also wonders about (relative) intonation with such a mute.

If the mutes in the Prague collection could be accurately dated to the eighteenth century, then perhaps it would be safe to conclude that they were intended to sharpen the pitch by a semitone on trumpets with the more modern bell contour. But where then are the mutes that must have been used by trumpeters of the era of Monteverdi and Fantini?

Acknowledgments

Many thanks are due to Robert Barclay of Gloucester, Ontario, for supplying the trumpet dimensions, and to Fred Holmgren of Athol, Massachusetts, for the loan of his mutes. Both also supplied much helpful information about the practicalities of Baroque trumpet manufacture and playing.

Appendix A: Modelling the Trumpet and Mute

The purpose of any scientific model is to make a complicated problem tractable by simplifying it while retaining enough of the original problem that the model produces valid information.

The propagation of sound in air is described by a differential equation called the wave equation. Essentially, the wave equation tells how a very small volume of air is moved and compressed by forces exerted on it by its neighbors. Ideally, one would analyze the behavior of sound within a flaring horn by solving the wave equation in all three space dimensions. This is practicable only for a limited number of shapes, none of which closely resembles a trumpet. Fortunately, brass instruments are much longer than they are wide, so that the diameter of an instrument is much smaller than the wavelength of sound at frequencies throughout the playing range of the instrument. Since neither sound pressure nor velocity can vary appreciably over distances much less than a wavelength, we can obtain quite good results by assuming that sound travels only along the axis of the instrument. That is, we assume the existence of "wavefront surfaces," surfaces across the horn on which sound pressure and velocity are constant in both amplitude and phase. The three-dimensional wave equation then reduces to a differential equation in one space dimension, distance measured along the axis of the instrument. If one neglects internal losses (viscous and thermal damping at the walls of the horn), at radian frequency ω the resulting equation is

$$p'' + \frac{S'}{S}p' + k^2 p = 0$$

where primes indicate differentiation with respect to distance x measured along the axis of the instrument, $p(x)$ is the sound pressure and $S(x)$ the area of the wavefront as a function of x, $k = \omega/c$ is the wavenumber, and c is the speed of sound.

This equation is called the *Webster horn equation* in most acoustics texts, after the 1919 paper by A. G. Webster.[9] It actually dates from the eighteenth century, when it was independently derived by Euler, Lagrange, and Daniel Bernoulli.[10] Euler even synthesized a bell contour for a brass instrument by making some reasonable assumptions about what properties are desirable in a musical instrument. The resulting shape is closer to a horn bell than a trumpet.[11]

The area of the wavefronts $S(x)$ is usually taken to be the area of a plane cross-section of the instrument. This is a good assumption throughout most of a brass instrument, but is questionable near the mouth of a rapidly flaring bell. Benade and Jansson[12] have addressed the accuracy of this. After a careful theoretical and experimental analysis, they conclude that the calculation of resonances using the plane-wavefront assumption "consistently overestimates the resonance frequencies by a small amount" but is "sufficient to obtain resonances with 0.5% accuracy". Since 0.5% is less than one-tenth of a semitone and the purpose of the present paper is to investigate *changes* in resonance frequencies brought about by the introduction of a mute, the plane-wave calculation is used here.

However, internal losses cannot safely be neglected; as it stands, Equation 1 is unsuitable. The damping causes the phase velocity of sound within the instrument to increase with frequency to a small but musically significant degree throughout the playing range. This lowers the resonance frequencies from the values one would calculate without damping.

Losses are incorporated into the model by replacing Equation 1 with the so-called "telegrapher's equations", a pair of simultaneous first-order differential equations describing the behavior of sound pressure p and volume velocity U. Such equations are often used in the analysis of electrical transmission lines; a brass instrument may be considered as an acoustic transmission line.

$$p' = -(jwL + R)U$$
$$U' = -(jwC + G)p$$

where L is the series inertance (per unit length of the transmission line), R the series resistance, C the shunt compliance, and G the shunt conductance. All four quantities are functions of the cross-section S(x). The values of L, R, C, and G are taken from the paper by Keefe.[13] The inertance L represents the mass of the vibrating air and the compliance C its compressibility. The quantity R measures the loss of energy through friction between the moving vibrating air and the wall of the tubing; G measures vibratory energy lost through heat conduction between the tubing and air compressed (and thereby heated) or rarefied (and cooled) by its vibration. The telegrapher's equations reduce to the Webster equation if losses are neglected (*i. e.*, if $R = 0$ and $G = 0$) Both R and G are inversely proportional to the tube diameter, so the narrower parts of an instrument account for most of the internal losses.

The heights of the impedance peaks (and the depths of the intervening valleys) are determined by the fraction of acoustic energy put into the mouthpiece that is reflected back to the mouthpiece to reinforce (or cancel) the vibration. It is therefore important to account correctly for the amount of energy lost, either within the instrument or by radiation from the bell.

For a circular cylindrical tube of uniform cross-section, Equation 2 can be solved exactly. The computational scheme used here starts from this solution rather than attempting to solve Equation 2 directly for a flaring instrument.

The smoothly flaring instrument is replaced by a piecewise cylindrical contour, a series of uniform tubes of equal length, each tube matching at its midpoint the diameter of the original instrument , as illustrated in Figure 8. One can see that if the number of cylinders is large (that is, if each cylinder is sufficiently short), this "stairstep" contour can be made to approach the original as closely as desired. A *transmission matrix* relating sound pressure p and volume velocity U at the ends of each of these short cylinders can be found by solving Equation 2. The transmission matrix of the flaring section is then just the product of the transmission matrices of the individual cylinders.

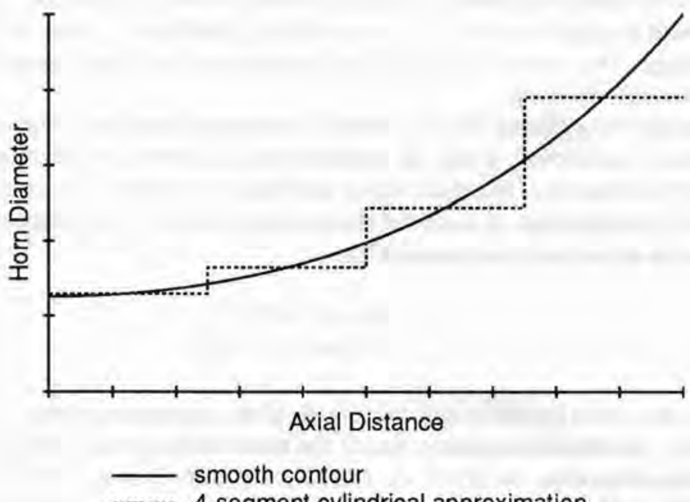

Figure 8

Piecewise cylindrical approximation of a smoothly flaring horn.

This method is relatively crude unless the number of cylinders is very large, but it can be refined using a technique of some antiquity called "deferred approach to the limit," popularized in the last quarter century or so in the Romberg method of numerical quadrature.[14] This works as follows. The error introduced by the stairstep approximation depends on the step size. In particular, it is known that the error is an *even* function of the length of each step. The transmission matrix of a tapered section is found for several "stairstep" horns with different numbers of steps, giving slightly different results. The differences between the different solutions can be used to estimate the error. The results are then combined in such a way as to remove most of the error inherent in the the piecewise cylindrical approximation.

For example, suppose F is some quantity we wish to find, say, the value of one of the elements of the transmission matrix. Suppose the length of each segment in a stairstep approximation is a and that the approximate solution is $C(a)$. Because the error is an even function of a, we can express the error as a power series with only even powers of a,

$$C\ (a) = F + c_2 a^2 + c_4 a^4 + c_6 a^6 + \cdots$$

where the values of the coefficients c_{2k} are as yet unknown.

If we evaluate $C(a)$ for two segment lengths $a = h$ and $a = 2h$, then

$$C\,(h) = F + c_2 h^2 + c_4 h^4 + c_6 h^6 + \cdots$$
$$C\,(2h) = F + 4c_2 h^2 + 16c_4 h^4 + 64c_6 h^6 + \cdots$$

The contribution of the h^2 term to the error can be removed without ever evaluating c_2 by multiplying the first of these equations by 4 and subtracting:

$$\frac{4C\ (h\ ) - C\ (2h\ )}{3} = F\ - 4c_4 h^4\ - 20c_6 h^6 - \cdots$$

If h is small enough, the left-hand side of Equation 5 will be a more accurate approximation of F than $C\,(h)$ and will have been obtained with only about 50% more labor. If $C\,(h)$ is evaluated for three different segment lengths, a similar procedure can be used to eliminate the h^4 error term as well as the h^2 error term, etc.

Convergence of this method is rapid. Comparison with an exact solution of Equation 1, where one is known (*e. g.*, for certain horn contours for lossless horns), shows that the numerical accuracy is better than is justified by the plane-wave approximation.

Using this method, the transmission matrix is then calculated independently for the various components of the instrument: the mouthpiece, the cylindrical tubing that constitutes more than two-thirds the length of the Baroque trumpet, and the bell (with and without a mute). The overall transmission matrix is found by multiplying the matrices of the components. Sound pressure and volume velocity at the input (mouthpiece) are thus related to those at the output (bell) by an equation of the form

$$\begin{bmatrix} p_{\text{in}} \\ U_{\text{in}} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} p_{\text{out}} \\ U_{\text{out}} \end{bmatrix}$$

where the four elements of the transmission matrix are computed as functions of frequency.

The bell end is terminated with a radiation impedance $Z_{rad} = p_{out} / U_{out}$. This impedance relating p_{out} and U_{out} accounts for the loss of sound energy into the room and for the inertia of the air just outside the bell of the trumpet. The transmission matrix then transforms the radiation impedance Z_{rad} to the acoustic input impedance $Z_{in} = p_{in} / U_{in}$ at the mouthpiece, which is the quantity we seek.

$$Z_{in} = \frac{H_{11} Z_{rad} + H_{12}}{H_{21} Z_{rad} + H_{22}}$$

The actual radiation impedance seen by a flaring horn is unknown; sound radiation from horns is not yet well understood theoretically. The radiation impedance used here is that of an unflanged circular tube of the same diameter as the end of the bell. This is known (by comparison with experiments on real instruments) to underestimate the amount of energy radiated, particularly for a rapidly flaring bell. For the muted trumpet, the radiation impedance is that for an unflanged tube whose diameter is the same as the hole bored through the mute. Internal losses and radiation loss both increase with frequency, but for the unmuted trumpet radiation loss increases much more rapidly above the playing range. Throughout the playing range, radiation loss is much less than internal viscous and thermal damping; thus, only for very high harmonics will the height of the calculated impedance peaks be appreciably higher than their actual values. (It is perhaps surprising that most of the acoustic energy produced by the player never escapes from the instrument as radiated sound, but is instead consumed by viscous and thermal damping at the walls of the tubing.)

This model serves well to calculate resonance frequencies, but ignores many subtleties that are important to the player and listener. For example, in a real trumpet, the thickness and softness of the metal and the bracing affect the tone quality, as do details of the shape of the mouthpiece.

The calculations were carried out on a Macintosh SE/30 computer. Mathematica (Wolfram Research) was used for the numerical modelling, FreeHand (Aldus) for drawing Figure 8, and Excel (Microsoft) for analyzing and smoothing the measured bell dimensions. The remaining figures were generated using software written by the author.

Appendix B: Contours of Mouthpiece, Trumpets, and Mute

Dimensions of mute and mouthpiece are from the examples supplied by Fred Holmgren. Dimensions of the trumpets are from drawings supplied by Robert Barclay. Although most of the dimensions are given here to a precision of 0.01 cm, this should not be construed as representing the accuracy of the measurements.

The mouthpiece

The mouthpiece has a hemispherical cup (diameter 19.0 mm) and a conical backbore (9.0 cm long, minimum diameter 4.3 mm, maximum diameter 9.2 mm). The full cup volume was used in the calculations, even though under playing conditions the player's lips would protrude into the cup, somewhat reducing its volume. Some of the calculations were repeated using two-thirds of the cup volume; the computed air-column resonance frequencies were slightly higher than with the full cup volume, but the intervals between them were essentially unchanged.

The trumpets

The trumpet is comprised of three sections: a cylinder made up of the two straight tubes and the two bows, a cone between the second bow and the boss, and the bell flare between the boss and the bell rim. (The boss is a sleeve, often decorated with an ornate knob, that covers the joint between the cone and the bell flare. The internal bore is smoothly flaring and does not follow the external contour of the knob.)

Table 1 gives the dimensions of the cylindrical and conical sections for the two trumpets.

	Trumpet	
Size in cm of	Hainlein	Ehe
cylinder length	163.77	164.35
cylinder diameter	1.05	1.10
cone length	28.98	36.00
cone diameter (small end)	1.05	1.10
cone diameter (large end)	1.57	1.36

Table 1
Dimensions of cylindrical and conical sections.

For the bell flare (the only part of the trumpets with a curving contour), the diameter of the bore was measured every 1.27 cm (0.5 inch) from the bell rim to the boss. These numbers, with their inevitable errors, were entered into a spreadsheet on the computer. First and second differences were calculated to reveal irregularities in the measurements. The measured values were then adjusted to smooth the contour. A cubic polynomial was used to interpolate values of the diameter between the measured points. For all points other than those in the final segments at the ends, the cubic passed through two points on either side of the interpolated point. Obviously, at the ends, it was necessary to have one point (the end diameter) on one side and three points on the other.

Distance from Bell Rim (cm)	Bell Flare Diameter (cm) Hanlein	Ehe	Distance from Bell Rim (cm)	Bell Flare Diameter (cm) Hanlein	Ehe
0.00	9.65	10.61	20.32	2.56	11.87
1.27	8.17	7.25	21.59	2.47	1.78
2.54	6.87	5.34	22.86	2.38	1.70
3.81	5.90	4.16	24.13	2.29	1.62
5.08	5.22	3.53	25.40	2.21	1.55
6.35	4.64	3.13	26.67	2.12	1.49
7.62	4.17	2.92	27.94	2.04	1.44
8.89	3.80	2.76	29.21	1.96	1.40
10.16	3.54	2.64	30.48	1.89	1.38
11.43	3.38	2.53	31.75	1.82	1.35
12.70	3.23	2.42	33.02	1.75	
13.97	3.09	2.33	34.29	1.69	
15.24	2.97	2.23	35.56	1.64	
16.51	2.85	2.14	36.83	1.60	
17.78	2.74	2.04	38.10	1.57	
19.05	2.65	1.95	39.37	1.55	

Table 2
Diameter of bell flares.

The length of the bell flare from the center of the boss to the bell rim was 38.33 cm for the Hainlein trumpet and 31.30 cm for the Ehe. Table 2 gives the values used in the interpolation. Note that the last value (farthest from the bell rim) is slightly beyond the length given above. This was to facilitate the interpolation by having equally spaced samples. The computer program switched from the bell flare to the cone at the stated length.

The mute

The outside diameter of the mute where it meets the bell is 3.10 cm. The internal diameter of the cup at that end (the upstream end) is 2.95 cm. The cup was taken to have an elliptical contour; the depth of the cup (major semiaxis of the ellipse) is 2.71 cm. The diameter of the hole bored through the mute is 0.635 cm and its length is 10.6 cm. The cup at the downstream end of the mute was ignored. While it may have a slight effect on the tone color of the muted trumpet, its effect on resonance frequencies is negligible.

NOTES

1. J. Keller, "Antique Trumpet Mutes," *Historic Brass Society Journal* 2 (1990): 97–103; and D. L. Smithers, "Antique Trumpet Mutes: A Retrospective Commentary," *Historic Brass Society Journal* 2 (1990): 104–111.

2. See A. H. Benade, *Fundamentals of Musical Acoustics* (London: Oxford University Press, 1976; reprinted with corrections, New York: Dover,1990), especially Chapter 20.

3. The word *harmonic*, perhaps unfortunately, has two quite distinct meanings when referring to brass instruments. Each note of the instrument is called a harmonic, as in "Middle C is the fourth harmonic of the natural trumpet, but the second harmonic of the valve trumpet". But each note, when sounded, produces a tone that is comprised of a fundamental at the playing frequency and harmonics at exact integer multiples of that frequency, as in "The third harmonic of middle C is a twelfth higher at the G atop the staff." The writer hopes that the context makes clear which meaning is intended.

4. H. Bouasse, *Instruments à Vent*, Librairie Delagrave (Paris, Vol. 1, 1929, Vol. 2, 1930).

5. Robert Pyle, "Factitious Tones and Hand-Stopping," *The Horn Call*, Vol. XXI, No. 1 (October 1990): 36–43 .

6. Like muting the Baroque trumpet, handstopping the horn lowers all the air-column resonance frequencies of the instrument. When the degree of stopping is slight, the intervals between notes are not much perturbed and the player perceives the change as a flattening of the instrument. With full stopping, the intervals more closely match the harmonic series of a shorter instrument which the player perceives as sharpening the instrument. There is no abrupt transition from one regime to the other, except perhaps in the player's mind. Backus (see J. Backus, "Input impedance curves

for the brass instruments," *Journal of the Acoustical Society of America* 60 (1976): 470–480) performed some experiments on the horn wherein the second resonance peak decreased in height as the degree of stopping increased. In the fully-stopped horn it was so heavily damped that it virtually disappeared. Backus claimed that this justified the renumbering of higher modes. However, earlier experiments by this writer (see Robert Pyle, "Pitch change of the stopped French horn," *Journal of the Acoustical Society of America* 36 (1964): 1034 (Abstract only), cited by Backus) showed a very clear second peak, though somewhat reduced in height as in Figure 3. It appears to this writer that the vanishing of the second peak in Backus's experiment is either due to an artifact of his experimental apparatus or to the use of a rather unusual form of stopping mute. More recent computer simulation of the stopped horn (see Pyle, "Factitious Tones and Hand-Stopping:" 36–43) produced results that agreed well with the earlier experiments of the author (see Robert Pyle, "Pitch change of the stopped French horn:" 1034). Pulse reflection measurements show that in the fully-stopped horn, the principal reflection that returns to reinforce the lip vibration comes from the position where the mute or hand meets the bell (the *upstream* end of the mute; [see Robert Pyle, "A time-domain study of the stopped horn," *Journal of the Acoustical Society of America*, 65 (1979), S73 (Abstract only): 11; and Pyle, "Factitious Tones and Hand-Stopping:" 36–43]). Thus it is not incorrect for the hornplayer to claim that full stopping has shortened the horn.

7. Benade, *Fundamentals of Musical Acoustics*, 411.

8. This statement is based in part on experimental measurements on modern trumpets with bells similar to the Ehe and a British-style duty bugle with a bell contour not unlike the Hainlein.

9. A. G. Webster, "Acoustical Impedance and the Theory of the Phonograph," *Proc. Natl. Acad. Sci.* (U.S.) 5 (1919): 275–282 .

10. See E. Eisner, "Complete Solutions of the 'Webster' Horn Equation," *Journal of the Acoustical Society of America* 41 (1967), 1126–1146, for a history of the repeated derivations of the "Webster" equation.

11. L. Euler, "Part Four [of a Treatise on Fluid Mechanics]: On the Motion of Air in a Tube" (in Latin), *Novi Comment. Acad. Sci. Petrop.* 16 (1771), Chapter 4: "On Very Small Vibrations of Air in a Tube of Nonuniform Width," 349–425. Reprinted in *Leonhardi Euleri Opera Omnia, Series* II, 13 (Zürich: Orell Füssli, 1955. Distributed by Springer Verlag, Berlin), 262–369.

12. A.H. Benade and E.V. Jansson, "On plane and spherical waves in horns with nonuniform flare," *Acustica* 31 (1974): 80-98.

13. D. H. Keefe, "Acoustical wave propagation in cylindrical ducts: Transmission line parameter approximations for isothermal and nonisothermal boundary conditions," *Journal of the Acoustical Society of America* 75 (1984): 58–62.

14. A. Ralston and H. Wilf, eds., *Mathematical Methods for Digital Computers,* ed., Vol. II, (New York: Wiley, 1967).

Robert Pyle first became interested in musical acoustics when his high-school physics teacher lent him Dayton Miller's book, "The Science of Musical Sounds". He pursued this interest through college and graduate school, eventually writing a doctoral dissertation in applied physics about horns, albeit solid horns for ultrasonics rather than the musical variety. After playing a variety of woodwind and brass instruments through high school, he settled on the horn, which he plays in community orchestras and bands. Since 1965, Dr. Pyle has worked in both acoustics and computer activities at Bolt, Beranek, and Newman in Cambridge, Massachusetts, where he currently earns his living by analyzing the performance of wide-area computer networks. Dr. Pyle is the acoustics editor of "The Horn Call" (the journal of the International Horn Society), and he has served several terms on the Technical Committee for Musical Acoustics of the Acoustical Society of America.